# Partition Studies. VII. Molecular Complex Formation in Fractional Extraction. Silver and Cineole Complexes of Cresols 

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The formation of silver and cineole molecule complexes of the isomeric cresols has been demonstrated, and the equilibrium constants for these reactions have been determined by a partition method. Equations relating partition coefficients and equilibrium constants for different types of complexes were derived in order to investigate the applicability of complex formation in separations by fractional extraction. The conditions under which complex formation enhanced separability of compounds were defined mathematically.

The possibility of utilizing molecular complexes for separation of organic compounds by the various fractional extraction procedures ${ }^{1}$ has hitherto received little attention. In this communication, the results of such an investigation are reported. The conditions under which the use of complex formation aids in the separation of closely related compounds have been defined in terms of partition coefficients and equilibrium constants. The validity of the theoretical treatment was tested by the measurement of the partition coefficients of the isomeric cresols in the presence of cineole and silver ion. From these data, the equilibrium constants for the formation of the complexes were calculated.

These reagents and the solvent system (cyclo-hexane-water) were chosen because they permitted adjustment of partition coefficients to values near unity. This is important because maximum separation of a pair of components by countercurrent extraction is obtained when the geometric mean of their partition coefficients is one. ${ }^{2,3}$

$$
\begin{gather*}
k_{\mathrm{B} \text { opt. }}=\sqrt{1 / R}  \tag{1}\\
\sqrt{k_{\mathrm{A} \text { opt. }} k_{\mathrm{B} \text { opt. }}}=1 \tag{2}
\end{gather*}
$$

Relation between Partition Coefficient and Equilibrium Constant of Complex Formation.-Equations relating equilibrium constants and partition coefficients for several types of complexes were derived by the method described in detail below for a $1: 1$ complex. Although expressions equivalent to equations 3,8 and 9 were derived previously by Winstein and Lucas, ${ }^{4}$ the present treatment demonstrates particularly the effect of complex formation on partition properties and hence is more useful for predicting the separability of compounds.

Case A: Formation of a 1:1 Complex.-It is assumed that the simple distribution law is valid and that a complexing agent, $Y$, reacts with a substance, $X$, in either an aqueous phase (I) or an immiscible non-polar organic phase (II) to
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E. T. Stiller, This Journal, 71, 2452 (1949); (f) C. Golumbic, A nal.
Chem., 28, 1210 (1951).
(2) (a) W. J. D. van Dijck and A. Schaafsma, U. S. Patent $2,245,945$
(1941); (b) S. Stene, Arkiv Kemi Mineral. Geol., 18H, No. 18 (1944).
(3) This relationship also applies to countercurrent distribution.
In that procedure, position $N$ of the maximum of a distribution band
is governed by partition coefficient $k$, and the total number of transfers
$n$, according to the equation, ${ }^{1 \mathrm{~b}} N=n k / k+1$. The optimum choice
of partition coefficients for maximum separation of a pair of com-
pounds, $A$ and $B$, whose $k_{A} / k_{B}=R$ is determined as
$\Delta N=N_{\mathrm{A}}-N_{\mathrm{B}}=n\left[k_{\mathrm{A}} /\left(k_{\mathrm{A}}+1\right)-k_{\mathrm{B}} /\left(k_{\mathrm{B}}+1\right)\right]=$
$n\left[R k_{\mathrm{B}} /\left(R k_{\mathrm{B}}+1\right)-k_{\mathrm{B}} /\left(k_{\mathrm{B}}+1\right)\right]$
$\left(\mathrm{d} \Delta N / \mathrm{d} k_{\mathrm{B}}\right)_{\max }=n R /\left(R k_{\mathrm{B}}+1\right)^{2}-n /\left(k_{\mathrm{B}}+1\right)^{2}=0$
(4) S. Winstein and H. J. Lucas. This Journal, 60, 836 (1938).
form a complex XY. The following notations are employed for the instance in which the complexing agent is soluble only in the aqueous phase:

$$
\begin{aligned}
& C_{\mathrm{II}}=\mathrm{X}_{1 \mathrm{I}}=\text { molar concentration of } \mathrm{X} \text { in phase II } \\
& C_{\mathrm{x}}=\mathrm{X}_{\mathrm{I}}+(\mathrm{XY})_{\mathrm{I}}=\text { total molar concentration of } \mathrm{X} \text { (free } \\
& \text { and combined) in phase I } \\
& C_{\mathrm{y}}=\mathrm{Y}_{1}+(\mathrm{XY})_{1}=\text { total molar concentration of } \mathrm{Y} \text { (free } \\
& \text { and combined) in phase I } \\
& \quad k=\frac{\mathrm{X}_{\mathrm{II}}}{\mathrm{X}_{\mathrm{I}}}=\frac{\mathrm{C}_{\mathrm{II}}}{\mathrm{X}_{1}}=\text { partition coefficient of free } \mathrm{X} \\
& \quad K=\frac{(\mathrm{XY})_{I}}{\mathrm{X}_{1} \cdot \mathrm{Y}_{\mathrm{I}}}=\text { complex formation constant }
\end{aligned}
$$

The apparent partition coefficient, $k^{\prime}$, which is measured in the presence of the complexing agent will then be

$$
\begin{gather*}
k^{\prime}=\frac{C_{11}}{C_{\mathrm{x}}}=\frac{\mathrm{X}_{\mathrm{II}}}{\mathrm{X}_{\mathrm{I}}+(\mathrm{XY})_{\mathrm{I}}}=\frac{\mathrm{X}_{\mathrm{II}}}{\mathrm{X}_{\mathrm{I}}+\mathrm{KX}_{\mathrm{I}} \mathrm{Y}_{1}}=\frac{k}{1+\mathrm{K} \mathrm{Y}_{\mathrm{I}}} \\
k^{\prime}=\frac{k}{1+K\left(C_{\mathrm{y}}-C_{\mathrm{x}}+\frac{\mathrm{C}_{1 \mathrm{I}}}{\mathrm{k}}\right)} \tag{3}
\end{gather*}
$$

When $C_{y} \gg(\mathrm{XY})_{\mathrm{I}}$ or $Y_{1} \gg(\mathrm{XY})_{\mathrm{I}}$, equation (3) is reduced to

$$
\begin{equation*}
k / k^{\prime}-1=K C_{y} \tag{4}
\end{equation*}
$$

Accordingly, a plot of $k / k^{\prime}-1$ as a function of $C_{y}$ should be a straight line passing through the origin, if a $1: 1$ complex is formed in the presence of excess complexing agent. The slope of this line gives the value of $K$, which is independent of the concentration of the complexing agent.

If Y is soluble only in non-aqueous phase II instead of phase $I$, the term $k / k^{\prime}$ of equation 4 is inverted. If, in addition, $X$ is a weak acid, the equilibria involved in the distribution are further complicated by ionization in the aqueous phase. Under these circumstances, it may be readily shown that the following equation applies

$$
\begin{equation*}
k^{\prime} / k\left[1+K_{\mathbf{i}} /\left(\mathrm{H}^{+}\right)\right]-1=K C_{y} \tag{5}
\end{equation*}
$$

in which $K_{i}$ is the ionization constant of the acid. If $K_{i}$ is known, and if the hydrogen ion concentration is fixed by a buffer, $K$ may be readily determined. If the $p \mathrm{H}$ of the buffer is sufficiently high so that the acid is essentially completely ionized, then equation 5 is reduced to

$$
\begin{equation*}
\left[k^{\prime} K_{i} / k\left(\mathrm{H}^{+}\right)\right]-1=K C_{y} \tag{6}
\end{equation*}
$$

Case B: Simultaneous Formation of $1: 1$ Complexes with Different Complexing Agents.-If, in addition to the reaction $\mathrm{X}+\mathrm{Y} \rightleftarrows \mathrm{XY}$ in phase I , a second, independent complexing reaction takes place in phase II, $X+Z \rightleftarrows X Z$, the partition coefficient, $k^{\prime}$, measures the quotient
$k^{\prime}=\left[\mathrm{X}_{\mathrm{II}}+(\mathrm{XZ})_{\mathrm{II}}\right] /\left[\mathrm{X}_{\mathrm{I}}+(\mathrm{XY})_{\mathrm{I}}\right]=$

$$
\begin{equation*}
k\left(1+K^{\prime} Z_{\mathrm{II}}\right) /\left(1+K Y_{\mathrm{I}}\right) \tag{7}
\end{equation*}
$$

where $K^{\prime}$ represents the equilibrium constant for the formation of $X Z$. $Y_{I}$ and $Z_{\text {II }}$ of equation 7 may be replaced by $C_{y}$ and $C_{z}$, respectively, when $C_{y} \gg$ (XY) ${ }_{\mathrm{I}}$ and $C_{2} \gg$ (XZ) ${ }_{\text {II }}$.

Case C: Formation of $1: 2$ and 2:1 Complexes in One Phase.-If the value of $K$ calculated by equation 4 increases with increasing values of $C_{y}$, the formation of a $1: 2$ complex is indicated in addition to the $1: 1$ complex

$$
\begin{aligned}
\mathrm{X}+\mathrm{Y} \longleftrightarrow \mathrm{XY} & K_{1}=\mathrm{XY} / \mathrm{X} \cdot \mathrm{Y} \\
\mathrm{XY}+\mathrm{Y} \longleftrightarrow \mathrm{XY}_{2} & K_{2}=\mathrm{XY}_{2} / \mathrm{Y} \cdot \mathrm{XY}
\end{aligned}
$$

From material balance considerations, it can be slown that

$$
\begin{equation*}
\left(1 / F_{1}\right)\left(k / k^{\prime}-\cdots 1\right)=K_{1}+K_{1} K_{2} Y_{I} \tag{8}
\end{equation*}
$$

if the complexing reaction is confined to phase I. If $Y_{1} \sim$ $C_{\mathrm{y}}, K_{1}$ and $K_{2}$ may be evaluated from a plot of $\frac{1}{C_{\mathrm{y}}}\left(\frac{k}{k^{\prime}}-1\right)$ us. $C_{y}$; otherwise, a method of successive approximations may be employed.

Conversely, when the calculated value of $K$ decreases with increasing concentration of $C_{y}$, a $2: 1$ complex is indicated

$$
X Y+X \longleftrightarrow X_{2} Y \quad K_{2}^{\prime}=X_{2} Y / X Y \cdot X
$$

and equation 9 applies.

$$
\begin{equation*}
\left(1 / K_{I}\right)\left(k / k^{\prime}-1\right)=K_{1}+2 X_{I} K_{1} K_{2}^{\prime} \tag{9}
\end{equation*}
$$

$K_{1}$ and $K_{2}^{\prime}$ may be calculated in a manner sinilar to that for $K_{1}$ and $K_{2}$ in equation 8 .

Silver Complexes of Cresols.-The data presented in Table I show the effect of increasing concentration of silver ion on the distribution of the isomeric cresols in the system cyclohexane-water. In all distributions, the silver ion concentration was much greater than the total concentration of phenol ( $0.00463 M$ ). The $k$ values are the partition coefficients of the cresols for distribution between cyclohexane and 1 molar potassium nitrate solution. Ionic strength of the silver nitrate solutions was maintained at unity by addition of appropriate amounts of potassium nitrate. Plots of the data according to equation 4 demonstrated that each cresol formed a $1: 1$ complex with silver ion over the concentration range studied. The equilibrium constants derived from the slopes ( $m$ ) of these curves are summarized in Table 1 .

## Tablef I

Distribution of Cresols between Cycloheyane and Water in the Presence of Silyer Nitrate at $25^{\circ 0}$


- In each distribution, the total concentration of cresol in the system was 0.00463 mole per liter. Whe aqueous phase was I molar potassimm nitrate.
complexes are largely determined by the relative inductive effects of nuclear substituents. A strong steric factor is also manifest among benzene derivatives containing large, bulky groups or many small ones, ${ }^{7}$ possibly because the silver ion, as demonstrated by Mulliken ${ }_{1}{ }^{8}$ is located in a position intermediate between the plane of the ring and its sixfold symmetry axis.

Both steric and electronic effects appear to be needed to explain the relative stability of the silver complexes of the cresols. Winstein and Lucas ${ }^{4}$ found that the argentation constant of phenol was 2.19 at $25^{\circ}$. The greater stability of the $o$-cresol complex may be attributed to slight enhancement of the basicity of the ring by the electron-releasing methyl group. A meta methyl group would be expected to be more effective; however, meta and para alkyl substituents may offer a progressively greater barrier to the approach of the silver ion to the benzernoid nucleus and thereby decrease the tendency for complex formation. An apparently related observation has been made with the xylenes ${ }^{\circ}$; $p$-xylene has a smaller argentation constant than $o$-xylene, $m$-xylene and toluene.

Cineole Complexes.-The effect of increasing concentrations of cineole on the distribution of cresols between cyclohexane and water is shown by the results presented in Table II. The $k$ values are the partition coefficients measured in the absence of cineole. Since ionization in the aqueous phase is negligible, these values essentially represent the distribution constants of the un-ionized phenols. In columns $5-7$ of Table II, data are presented for the distribution of o-cresol between cyclohexane and a phosphate buffer of $p \mathrm{H} 11.08$. The $k$ values for this series are those obtained in the distribution of the cresols between cyclohexane and a slightly acidic phosphate buffer. ${ }^{10}$

Plots of the experimental data showed that cineole formed a complex of the 1:1 type with each cresol. The equilibrium constants for the formation of the cineole complexes are given in Table II.

Cineole contains a strongly basic oxygen atom ${ }^{11}$ and is known to form complexes with a large varicty of proton donors. Its complexing action

Table II
Distribution of Cresols between Cyclohexane and Water in the Presence of Cineole at $25^{\circ}$

| Gole/fiver | H | $o \text {-Crese? }$ | K | k | o-Cresolb | K | $k$ | m-Cresol | $K$ | $k$ | $\underset{k^{\prime}}{p-\mathrm{Cresol}}$ | K |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1.26 | ... | . | $2.00^{c}$ |  | . | 0.66 | . . . | . | 0.68 |  |  |
| 0.10 | . | 2.97 | 13.6 | . . . | 0.56 | 13.4 | . . | $\ldots$ | . | . . | $\cdots$ | . |
| . 25 |  | 5.41 | 13.2 | $\cdots$ | 1.06 | 13.8 | . | 3.35 | 16.3 | . | 3.55 | 16.8 |
| . 50 |  | 9.87 | 13.7 | . | 1.79 | 13.0 | . | 6.14 | 16.6 | . | 6.35 | 16.7 |
| . 75 |  | 13.7 | 13.2 | . . | . . | . . |  | 9.42 | 17.7 | . | 9.00 | 16.3 |
| 1.00 |  | 17.5 | 12.9 | . $\cdot$ | $\cdots$ | . |  | 12.2 | 17.5 | . | 12.2 | 17.0 |
|  |  | $m=13$ |  |  | $m=13$ |  |  | $m=17$ |  |  | $m=17$ |  |

a In each distribution, the total concentration of cresol in the system was 0.00463 mole per liter. $b$ Unless otherwise noted, the aqueous phase was $0.5 M$ phosphate buffer of $p H 11.08$. "The aqueous phase was phosphate buffer of $p H 6.65$.

Silver complexes of aromatic compounds appear to be formed by a Lewis acid-base interaction in which the $\pi$-electrons of the aromatic ring function as electron donors to the silver ion. ${ }^{5}$ According to Andrews and Keefer, ${ }^{6}$ the relative stabilities of the

[^0]with cresols may be assumed to be a Lewis acidbase interaction, with a hydrogen bond serving as the actual linkage between the molecules. The
(7) L. J. Andrews and R. M. Keefer, ibid., 72, 5034 (1950).
(8) R. S. Mulliken, J. Chem. Phys., 19, 514 (1951).
(9) L. J. Andrews and R. M. Keefer, This Journal, 71, 3644 (1949).
(10) C. Golumbic, M. Orchin and S. Weller, ibid., 71, 2624 (1949).
(11) S. Searles and M. Tamres, ibid., 73, 3704 (1951),
relative stabilities of the complexes are in accord with the relative hydrogen bonding tendencies of the phenols, ${ }^{9}$ namely, $o<p, m$.

It is noteworthy that the $o$-cresol complex, which is least stable, is the one easiest to prepare in the pure state. This complex crystallizes from a mixture of isomeric cresols and can be used for the quantitative determination of $o$-cresol. ${ }^{12}$

Molecular Complexes in the Separation of Com-pounds.-In order to separate any pair of substances (A and B) by extraction, the ratio of their partition coefficients, or $\beta$-values, must be different from unity. In the presence of a complexing agent, the ratio of observed partition coefficients, $k_{\mathrm{A}}^{\prime} / k_{B}^{\prime}$ is given by the expression

$$
\begin{equation*}
k_{\mathbf{A}}^{\prime} / k_{\mathbf{B}}^{\prime}=k_{\mathbf{A}}\left(1+Y_{1} K_{\mathbf{B}}\right) / k_{\mathbf{B}}\left(1+Y_{1} K_{\mathbf{A}}\right) \tag{10}
\end{equation*}
$$

for the case where a $1: 1$ complex is formed in phase I. Without loss of generality, let $k_{\mathrm{A}}>k_{\mathrm{B}}$. The ratio $k_{A}^{\prime} / k^{\prime}{ }_{\mathrm{B}}$ is greater than the corresponding ratio $k_{\mathrm{A}} / k_{\mathrm{B}}$ when $\left(1+Y_{\mathrm{I}} K_{\mathrm{B}}\right) /\left(1+Y_{\mathrm{I}} K_{\mathrm{A}}\right)>1$. (The latter term is inverted if complexing occurs in phase II.) The ratio $k_{\mathrm{A}}^{\prime} / k^{\prime} \mathrm{B}$ changes with increase in concentration of complexing agent and approaches the limiting value given in the equation

$$
\begin{equation*}
k_{\mathrm{A}}^{\prime} / k_{\mathrm{B}}^{\prime}=k_{\mathrm{A}} K_{\mathrm{B}} / k_{\mathrm{B}} K_{\mathrm{A}} \tag{11}
\end{equation*}
$$

The limiting expression, if complete complexing in phase II is accompanied by complete ionization in phase $I$, is

$$
\begin{equation*}
k_{\mathrm{A}}^{\prime} / k_{\mathrm{B}}^{\prime}=k_{\mathrm{A}} K_{\mathrm{A}}\left(K_{\mathrm{i}}\right)_{\mathrm{B}} / k_{\mathrm{B}} K_{\mathrm{B}}\left(K_{1}\right)_{\mathrm{A}} \tag{12}
\end{equation*}
$$

Thus, the $\beta$-value, $k_{\mathrm{A}} / k_{\mathrm{B}}$, of 1.03 for a mixture of $p$ and $m$-cresol can be increased to a maximum of $(0.83) \times(2.1) /(0.81) \times(1.5)=1.44$ by complexing with silver ion. The separation of these isomers now becomes feasible by a fractional extraction procedure, such as countercurrent distribution, ${ }^{16}$ provided that $C_{y} \gg \mathrm{XY}$. This latter requirement should be fulfilled in order to maintain a linear partition isotherm which is desirable in countercurrent distribution. For a mixture of $o$-cresol with $m$ - or $p$-cresol, it can be shown similarly that complexing with silver decreases the $\beta$-value and is thus useless for separation of these pairs of isomers.

Similarly, the formation of cineole complexes does not increase separation of the cresols and actually nullifies in part the increase in $\beta$-value that results from ionization alone. ${ }^{9}$
It is of interest to examine possibilities for separation when the substances being extracted react with a different complexing agent in each phase (case B). The limiting expression for the $\beta$-value under these conditions is given by the equation

$$
\begin{equation*}
k_{\mathrm{A}}^{\prime} / k_{\mathrm{B}}^{\prime}=k_{\mathbf{A}} K_{\mathrm{A}}^{\prime} K_{\mathbf{B}} / k_{\mathrm{B}} K_{\mathrm{B}}^{\prime} K_{\mathrm{A}} \tag{13}
\end{equation*}
$$

where $K$ and $K^{\prime}$ represent the equilibrium constants for the formation of complexes in phase I and II, respectively. A calculation of $k^{\prime} / k^{\prime} k_{B}$ according to this equation shows that sinultaneous com-
(12) F. M. Potter and H. B. Williams, J. Soc. Chem. Ind., 51, 59 T (1932).
plexing with cineole and silver ion is no better and may be less effective than complexing with either reagent alone. This conclusion was verified by determining the partition coefficients of the individual cresols when distributed between 1 molar silver nitrate solution and cyclohexane containing different concentrations of cineole, The partition ratios to be expected under these circumstances can be calculated by means of equation 7 . As shown by the results summarized in Table III, excellent agreement was obtained between theory and experiment.

## Table III

Effect on Partition Cobfficient of Simultaneous Formation of Cineole and Silver Complexes of Cresols

| $\underset{M}{\mathrm{AgNO}_{2}}$ | Cineole, M | $\stackrel{O \text { Cresol }}{k^{\prime} \text { obsd. } \quad k^{\prime} \text { caled. } .}$ |  | $\underset{k_{\text {obsd. }}^{\prime}}{m \text {-Cresol }}$ |  | $\begin{array}{r} p \text {-Cresol } \\ \text { obed. } \quad k^{\prime} \text { ce } \end{array}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.0 | 0.25 | 1.9 | 2.0 | 1.4 | 1.4 | 1.7 | 1.7 |
| 1.0 | . 50 | 3.4 | 3.6 | 2.4 | 2.5 | 3.2 | 3.2 |

## Experimental

Materials.-The source and purity of the cresols used in this work are described elsewhere. ${ }^{9}$ The cineole was a commercial product which was purified by means of the resorcinol coinplex. ${ }^{13}$ The silver nitrate and potassium nitrate were of reagent-grade quality. Spectrographic-grade cyclohexane was employed as the organic base.

Methods.--The distributions were carried out in centrifuge separatory funnels by the technique described previously. ${ }^{10,14}$ A standard solution of the cresol $(0.5 \mathrm{mg}$. per inl.) was added to an equal volume of the aqueous phase in a $125-\mathrm{ml}$. separatory funnel and immersed in a water thermostat maintained at $25.00 \pm 0.02^{\circ}$. The phases were mixed by vigorous hand shaking for two-minute periods. The establishment of equilibrium was checked by the method of Barry, et al. ${ }^{15}$ The measurements of concentration were nade by ultraviolet spectroscopy at $280 \mathrm{~m} \mu$ with the Beckman quartz spectrophotometer. The partition coefficient was determined by the relation $k=C / C_{0}-C$ where $C_{0}$ and $C$ are the concentration of phenol in the cyclohexane before and after the equilibration. For the runs employing the highest cineole concentrations, the absorbence had to be corrected for the slight contribution of cineole. Pure cineole exhibits only slight general absorption in the ultraviolet region. Cineole is very sparingly soluble in water ( $0.2 \%$ ) and is almost completely removed from aqueous solution by extraction with cyclohexane.

In the concentration range involyed in this work, the partition coefficients of the cresols are known to be essentially independent of their concentration. ${ }^{16}$ To determine the accuracy of the partition measurements, recovery experiments were made in which both immiscible phases were analyzed after equilibration. The sum of the quantities of cresol in each layer thus obtained was always within $1 \%$ of the original amount. However, the accuracy of the equilibrium constants is no greater than about $10-15 \%$ because a small variation in partition ratio is reflected in a proportionally much greater change in the calculated equilibriun. constant.
Acknowledgment.-The authors are indebted to Dr. Milton Orchin for constructive criticism and to George Goldbach for technical assistance.
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[^0]:    (5) R. S. Mulliken, This Journal, 72, 600 (1950).
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    (14) L. C. Craig, J. Biol. Chem., 150, 33 (1943).
    (15) G. T. Barry, Y. Sato and L. C. Craig, ibid., 174, 209 (1948).
    (16) C. Golumbic, This Journal, 71, 2627 (1949).

